The first part of Chapter 4 of *Core Connections, Course 3* ties together several ways to represent the same relationship. The basis for any relationship is a consistent pattern that connects input and output values. This course uses tile patterns to help visualize algebraic relationships. (Note: In this course we consider tile patterns to be continuous relationships and graph them with a continuous line or curve.) These relationships may also be displayed on a graph, in a table, or as an equation. In each situation, all four representations show the same relationship. Students learn how to use each format to display relationships as well as how to switch from one representation to another. We use the diagram at right to show the connections between the various ways to display a relationship and call it the "representations web." See the Math Notes box in Lesson 4.1.7 of the *Core Connections, Course 3* text. For additional examples and practice see the *Core Connections, Course 3* Checkpoint 6 materials.
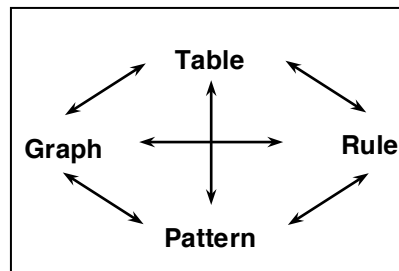
Example 1

At this point in the course we use the notion of growth to help understand linear relationships. For example, a simple tile pattern may start with two tiles and grow by three tiles in each successive figure as shown below.
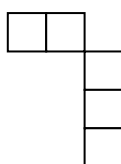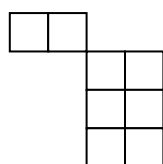

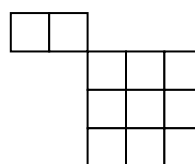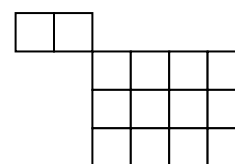Fig. 0   Fig. 1   Fig. 2   Fig. 3   Fig. 4

The picture of the tile figures may also be described by an equation in $y = mx + b$ form, where x and y are variables and m represents the growth rate and b represents the starting value of the pattern. In this example, $y = 3x + 2$, where 2 represents the number of tiles in the original figure (usually called "Figure 0") and 3 is the "growth factor" that describes the rate at which each successive figure adds tiles to the previous figure. This relationship may also be displayed in a table, called an "$x \rightarrow y$ table," as shown below. The rule is written in the last column of the table.

Figure number (x)	0	1	2	3	4	x
Number of tiles (y)	2	5	8	11	14	$3x + 2$

Finally, the relationship may be displayed on an *xy*-coordinate graph by plotting the points in the table as shown at right. The highlighted points on the graph represent the tile pattern. The line represents all of the points described by the equation $y = 3x + 2$.
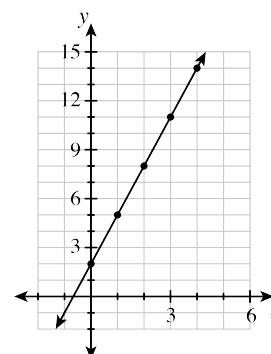
Example 2

Draw Figures 0, 4, and 5 for the tile pattern below. Use the pattern to predict the number of tiles in Figure 100, describe the figure, write a rule that will give the number of tiles in any figure, record the data for the first six tiles (Figures 0 through 5) in a table, and graph the data.
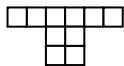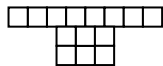

| Fig. 0 | Fig. 1 | Fig. 2 | Fig. 3 | Fig. 4 | Fig. 5 |

Each figure adds four tiles: two tiles to the top row and two tiles to the lower portion of the figure. Figure 0 has two tiles, so the rule is $y = 4x + 2$ and Figure 100 has $4(100) + 2 = 402$ tiles. There are 202 tiles in the top row and 200 tiles in the lower portion of figure 100. The table is:

Figure number (x)	0	1	2	3	4	5	x
Number of tiles (y)	2	6	10	14	18	22	$4x + 2$

The graph is shown at right.
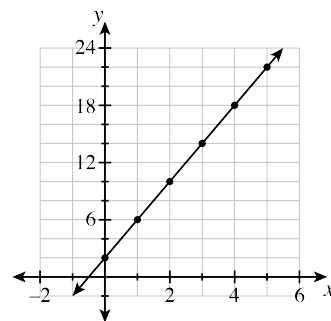
Example 3

Use the table below to determine the rule in $y = mx + b$ form that describes the pattern.

| input (x)	–2	–1	0	1	2	3	4	5	x
output (y)	–8	–5	–2	1	4	7	10	13	

+3 +3 +3 +3 +3 +3 +3

The constant difference between the output values is the growth rate, that is, the value of m. The output value paired with the input value $x = 0$ is the starting value, that is, the value of b. So this table can be described by the rule: $y = 3x - 2$. Note: If there is not a constant difference between the output values for consecutive integer input values, then the rule for the pattern is not in the form $y = mx + b$.

Example 4

Use the graph at right to create an $x \rightarrow y$ table, then write a rule for the pattern it represents.

First transfer the coordinates of the points into an $x \rightarrow y$ table.

input (x)	0	1	2	3	x
output (y)	5	1	–3	–7	
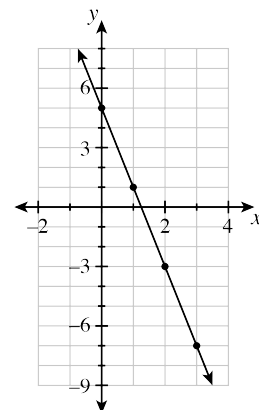
Using the method described in Example 3, that is, noting that the growth rate between the output values is –4 and the value of y at $x = 0$ is 5, the rule is: $y = -4x + 5$.

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Problems

1. Based on the tile pattern below, draw Figures 0, 4, and 5. Then find a rule that will give the number of tiles in any figure and use it to find the number of tiles in Figure 100. Finally, display the data for the first six figures (numbers 0-5) in a table and on a graph.
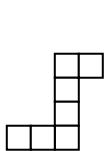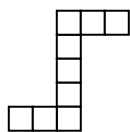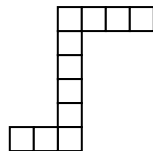

Fig. 0   Fig. 1   Fig. 2   Fig. 3   Fig. 4   Fig. 5

2. Based on the tile pattern below, draw Figures 0, 4, and 5. Then find a rule that will give the number of tiles in any figure and use it to find the number of tiles in Figure 100. Finally, display the data for the first six figures (numbers 0-5) in a table and on a graph.

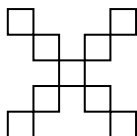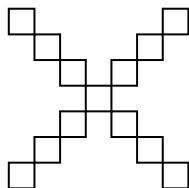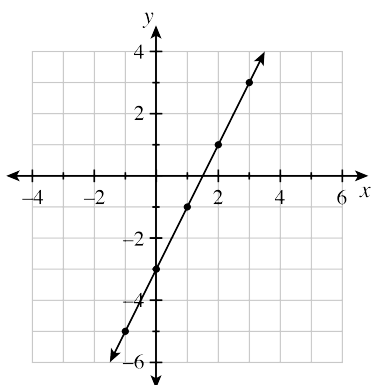Fig. 0   Fig. 1   Fig. 2   Fig. 3   Fig. 4   Fig. 5

Use the patterns in the tables and graphs to write rules for each relationship.
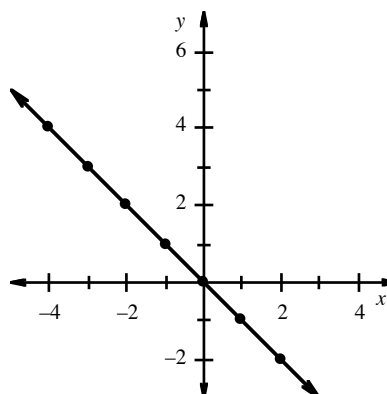
3.

input (x)	−3	−2	−1	0	1	2	3	4	5
output (y)	−11	−8	−5	−2	1	4	7	10	13

4.

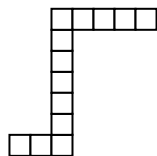| input (x)	−3	−2	−1	0	1	2	3	4	5
output (y)	10	8	6	4	2	0	−2	−4	−6

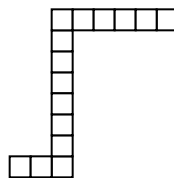5.

6.

Answers

1.

Fig. 0   Fig. 4   Fig. 5

The rule is $y = 2x + 5$. Figure 100 will have 205 tiles. It will have a base of three tiles, with 102 tiles extending up from the right tile in the base and 100 tiles extending to the right of the top tile in the vertical extension above the base.
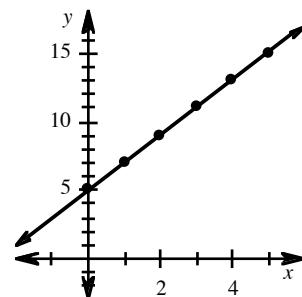
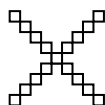| Figure number (x)	0	1	2	3	4	5
Number of tiles (y)	5	7	9	11	13	15

2.

Fig. 0   Fig. 4   Fig. 5
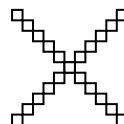
The rule is $y = 4x + 1$. Figure 100 will have 401 tiles in the shape of an "X" with 100 tiles on each "branch" of the X, all connected to a single square in the middle.
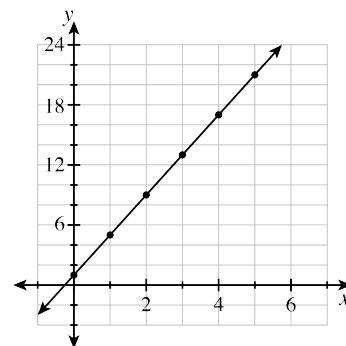
Figure number (x)	0	1	2	3	4	5
Number of tiles (y)	1	5	9	13	17	21

3. $y = 3x - 2$

4. $y = -2x + 4$

5. $y = 2x - 3$

6. $y = -x$

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